[2] $(3D-1)(7D-4)[x]+(3D-1)(5D-2)[y]=(3D-1)[15t^2]=3(30t)-15t^2 \overset{(2)}{=} 90t-15t^2$

$-(5D-2)(4D-2)[x]-(5D-2)(3D-1)[y]=-(5D-2)[9t^2]=-5(18t)+2(9t^2)=-90t+18t^2$ (2)

$$(21D^2-19D+4-(20D^2-18D+4))[x]=3t^2$$

ALL MARKED ITEMS (1) POINT UNLESS OTHERWISE INDICATED

(2) $(D^2-D)[x]=3t^2 \rightarrow x''-x'=3t^2$

$r^2-r=0 \rightarrow r(r-1)=0 \rightarrow r=0,1$

$x_h=c_1+c_2e^t$

$x_p = (2)\ t(A_1t^2+B_1t+C_1)$ (2)

$= A_1t^3+B_1t^2+C_1t$

$x_p' = 3A_1t^2+2B_1t+C_1$

$x_p'' = 6A_1t+2B_1$

$-x_p' \quad -3A_1t^2-2B_1t-C_1$

$\overset{(2)}{=} -3A_1t^2+(6A_1-2B_1)t+(2B_1-C_1)=3t^2$

$-3A_1=3 \rightarrow A_1=-1$

$-6-2B_1=0 \rightarrow B_1=-3$

$-6-C_1=0 \rightarrow C_1=-6$

$x_p=-t^3-3t^2-6t$

$x=-t^3-3t^2-6t+c_1+c_2e^t$

$$-(4D-2)(7D-4)[x]-(4D-2)(5D-2)[y]=-(4D-2)[15t^2]=-4(30t)+2(15t^2)=-120t+30t$$ (2)

$$(7D-4)(4D-2)[x]+(7D-4)(3D-1)[y]=(7D-4)[9t^2]=7(18t)-4(9t^2)=126t-36t^2$$ (2)

$$(-(20D^2-18D+4)+21D^2-19D+4)[y]=6t-6t^2$$

$$(D^2-D)[y]=6t-6t^2 \longrightarrow y_h=k_1+k_2e^t$$

$$y_p=A_2t^3+B_2t^2+C_2t$$

$$-3A_2=-6 \longrightarrow A_2=2$$ (1/2)

$$12-2B_2=6 \longrightarrow B_2=3$$ (1/2)

$$6-C_2=0 \longrightarrow C_2=6$$ (1/2)

$$y_p=2t^3+3t^2+6t$$

$$y=2t^3+3t^2+6t+k_1+k_2e^t$$

$$(4D-2)[x]+(3D-1)[y]=4x'-2x+3y'-y=9t^2$$

$$\begin{array}{lll} 4x' & =4(-3t^2-6t-6+c_2e^t) \text{ (2)} & \\ -2x & -2(-t^3-3t^2-6t+c_1+c_2e^t) & \\ +3y' & +3(6t^2+6t+6+k_2e^t) \text{ (2)} & \\ -y & -(2t^3+3t^2+6t+k_1+k_2e^t) & \end{array}$$

$$=\boxed{\begin{array}{l} -12t^2-\cancel{24t}-24-4c_2e^t \\ +\cancel{2t^3}+6t^2+\cancel{12t}-2c_1-2c_2e^t \\ +18t^2+\cancel{18t}+18+3k_2e^t \\ -\cancel{2t^3}-3t^2-\cancel{6t}-k_1-k_2e^t \end{array}}$$ (6)

$$=9t^2+(-k_1-2c_1-6)+(2k_2-6c_2)e^t=9t^2$$ (4 1/2)

$$-k_1-2c_1-6=0 \longrightarrow k_1=-2c_1-6$$ (2)

$$2k_2-6c_2=0 \longrightarrow k_2=3c_2$$ (2)

$$x=-t^3-3t^2-6t+c_1+c_2e^t$$ (1/2)

$$y=2t^3+3t^2+6t+(-2c_1-6)+3c_2e^t$$

[3] $r^3+r^2+9r+9=0$

$r^2(r+1)+9(r+1)=0$

$(r^2+9)(r+1)=0 \longrightarrow r=\pm 3i, -1 \longrightarrow y_h = c_1\cos 3x + c_2 \sin 3x + c_3 e^{-x}$ 2

$y''' + y'' + 9y' + 9y = 10xe^{-x}$

$y_p = x(Ax+B)e^{-x}$ 2 2

$= (Ax^2+Bx)e^{-x}$

$$y_p' = \boxed{\begin{array}{l}(2Ax+B)e^{-x}\\ +(-Ax^2-Bx)e^{-x}\end{array}}$$ 2

$= (-Ax^2+(2A-B)x+B)e^{-x}$

$$y_p'' = \boxed{\begin{array}{l}(-2Ax+(2A-B))e^{-x}\\ +(Ax^2+(-2A+B)x-B)e^{-x}\end{array}}$$ $4\frac{1}{2}$

$= (Ax^2+(-4A+B)x+(2A-2B))e^{-x}$

$$y_p''' = \boxed{\begin{array}{l}(2Ax+(-4A+B))e^{-x}\\ +(-Ax^2+(4A-B)x+(-2A+2B))e^{-x}\end{array}}$$ $4\frac{1}{2}$

$= (\cancel{-Ax^2}+(6A-\cancel{B})x+(-6A+3B))e^{-x}$

$$\begin{array}{l}+y_p''\\ +9y_p'\\ +9y_p\end{array} \begin{array}{|l}+(\cancel{Ax^2}+(-4A+\cancel{B})x+(2A-2B))e^{-x}\\ +(\cancel{-9Ax^2}+(18A-\cancel{9B})x+9B)e^{-x}\\ +(\cancel{9Ax^2}+\cancel{9B}x)e^{-x}\end{array}$$ $4\frac{1}{2}$

$= (20Ax+(-4A+10B))e^{-x} = 10xe^{-x}$

2

$20A=10 \longrightarrow A=\frac{1}{2}$

$-2+10B=0 \longrightarrow B=\frac{1}{5}$

$y_p = \left(\frac{1}{2}x^2+\frac{1}{5}x\right)e^{-x}$

$$y''' + y'' + 9y' + 9y = 37e^{-x}\cos 3x$$

$$y = \left(\tfrac{1}{2}x^2 + \tfrac{1}{5}x\right)e^{-x} + 2e^{-x}\cos 3x + \tfrac{1}{3}e^{-x}\sin 3x + c_1\cos 3x + c_2\sin 3x + c_3e^{-}$$

$$y_p = Ae^{-x}\cos 3x + Be^{-x}\sin 3x \quad 2$$

$$y_p' = -Ae^{-x}\cos 3x - 3Ae^{-x}\sin 3x + 3Be^{-x}\cos 3x - Be^{-x}\sin 3x \quad 2$$

$$= (-A+3B)e^{-x}\cos 3x + (-3A-B)e^{-x}\sin 3x$$

$$y_p'' = (A-3B)e^{-x}\cos 3x + (3A-9B)e^{-x}\sin 3x + (-9A-3B)e^{-x}\cos 3x + (3A+B)e^{-x}\sin 3x \quad 4\tfrac{1}{2}$$

$$= (-8A-6B)e^{-x}\cos 3x + (6A-8B)e^{-x}\sin 3x$$

$$y_p''' = (8A+6B)e^{-x}\cos 3x + (24A+18B)e^{-x}\sin 3x + (18A-24B)e^{-x}\cos 3x + (-6A+8B)e^{-x}\sin 3x \quad 4\tfrac{1}{2}$$

$$= (26A-18B)e^{-x}\cos 3x + (18A+26B)e^{-x}\sin 3x$$

$$+y_p'' \quad + (-8A-6B)e^{-x}\cos 3x + (6A-8B)e^{-x}\sin 3x$$

$$+9y_p' \quad + (-9A+27B)e^{-x}\cos 3x + (-27A-9B)e^{-x}\sin 3x \quad 4\tfrac{1}{2}$$

$$+9y_p \quad + (9A)e^{-x}\cos 3x + (9B)e^{-x}\sin 3x$$

$$= (18A+3B)e^{-x}\cos 3x + (-3A+18B)e^{-x}\sin 3x = 37e^{-x}\cos 3x \quad 2$$

$$18A + 3B = 37 \qquad -3A + 18B = 0$$

$$108B + 3B = 37 \longleftarrow A = 6B \quad 2$$

$$111B = 37$$

$$B = \tfrac{1}{3} \quad \tfrac{1}{2} \longrightarrow A = 2 \quad \tfrac{1}{2}$$

$$y_p = 2e^{-x}\cos 3x + \tfrac{1}{3}e^{-x}\sin 3x$$

[4][a] $y = \sin x \longrightarrow y' = \cos x \ (\tfrac{1}{2}) \longrightarrow y'' = -\sin x \ (\tfrac{1}{2})$

$$(\tan^2 x)(-\sin x) - 3\tan x \cos x + (3 + \tan^2 x)\sin x$$

$$= (\tfrac{1}{2}) \cancel{-\sin x \tan^2 x} \ \boxed{-3\frac{\cancel{\sin x}}{\cancel{\cos x}}\cancel{\cos x}} + \cancel{3\sin x} + \cancel{\sin x \tan^2 x} \ (\tfrac{1}{2}) = 0 \ (\tfrac{1}{2})$$

[b] $y_2 = v\sin x$

$y_2' = v'\sin x + v\cos x$

$y_2'' = v''\sin x + v'\cos x + v'\cos x - v\sin x = v''\sin x + 2v'\cos x - v\sin x$ (2)

$$\boxed{\begin{aligned} &(\tan^2 x)(v''\sin x + v'(2\cos x) - v\sin x) \\ -&(3\tan x)(\qquad v'\sin x + v\cos x) \\ +&(3+\tan^2 x)(\qquad v\sin x) \end{aligned}}$$ (4½)

$$= v''(\sin x\tan^2 x) + v'(2\overset{\sin x}{\cancel{\cos x}}\tan^{\cancel{2}} x - 3\sin x\tan x) + v\cancel{(-\sin x\tan^2 x - 3\cos x\tan x + 3\sin x + \sin x\tan^2 x)}$$ 0 (FROM [a]) (2)

$= v''\sin x\tan^2 x - v'\sin x\tan x = 0$

$v''\tan x - v' = 0$

$u = v' \longrightarrow \frac{du}{dx} = v''$

$(\tan x)\frac{du}{dx} - u = 0$

$\frac{du}{dx} = u\cot x$

$\int \frac{1}{u}\,du = \int \cot x\,dx$

(2) $\ln|u| = \ln|\sin x|$

$v' = u = \sin x$ (½)

$v = -\cos x \longrightarrow$ USE $v = \cos x$

$y_2 = \sin x\cos x$

[c] SWAP $y_1 = \sin x \cos x$, $y_2 = \sin x$

$$W = \begin{vmatrix} \sin x \cos x & \sin x \\ \cos^2 x - \sin^2 x & \cos x \end{vmatrix} = \sin x \cos^2 x - (\sin x \cos^2 x - \sin^3 x) = \sin^3 x$$

2 $\frac{1}{2}$

$$g = \frac{\sin x \cancel{\tan x}}{\tan^{\cancel{2}} x} = \sin x \cot x = \cancel{\sin x} \frac{\cos x}{\cancel{\sin x}} = \cos x$$

2

$$y_p = -\sin x \cos x \int \frac{\cos x \cancel{\sin x}}{\sin^{\cancel{3}\,2} x} dx + \sin x \int \frac{\cos x (\cancel{\sin x} \cos x)}{\sin^{\cancel{3}\,2} x} dx$$

$4\frac{1}{2}$

$$= -\sin x \cos x \int \csc x \cot x \, dx + \sin x \int \cot^2 x \, dx$$

$4\frac{1}{2}$

$$= -\sin x \cos x (-\csc x) + \sin x \int (\csc^2 x - 1) dx$$

$$= \cos x + \sin x (-\cot x - x)$$

$$= \cos x - \cos x - x \sin x$$

$$= -x \sin x$$

$$y = -x \sin x + C_1 \sin x \cos x + C_2 \sin x$$

[5] $x = \ln t \longrightarrow t = e^x$ (1/2)

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^x \frac{dy}{dt} \quad (2)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(e^x \frac{dy}{dt}\right) = e^x \frac{dy}{dt} + e^x \frac{d}{dx}\frac{dy}{dt} \quad (2)$$

$$= e^x \frac{dy}{dt} + e^x \left(\frac{d}{dt}\frac{dy}{dt}\right)\frac{dt}{dx} \quad (2)$$

$$= e^x \frac{dy}{dt} + e^x \left(e^x \frac{d^2y}{dt^2}\right) \quad (2)$$

$$= e^{2x}\frac{d^2y}{dt^2} + e^x \frac{dy}{dt} \quad (1/2)$$

$$9\frac{d^2y}{dx^2} + 3(2e^x - 3)\frac{dy}{dx} + e^{2x}y = 0$$

$$9\left(e^{2x}\frac{d^2y}{dt^2} + e^x\frac{dy}{dt}\right) + 3(2e^x - 3)e^x\frac{dy}{dt} + e^{2x}y = 0$$

$$9e^{2x}\frac{d^2y}{dt^2} + \cancel{9e^x\frac{dy}{dt}} + 6e^{2x}\frac{dy}{dt} - \cancel{9e^x\frac{dy}{dt}} + e^{2x}y = 0$$

$$e^{2x}\left(9\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + y\right) = 0$$

(1/2) $$9\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + y = 0$$